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Generalization of Multi-Level Programming Technique - A Brief Description

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Abstract

The separation of policy problems into two components has long been accepted as a rational approach. Multilevel programming is applicable in those cases in which a mathematical programming model describes the implicit behavioral set. It encompasses multiple levels of optimization, multilevel programming constitute a generalization of mathematical programming. The algorithm handles two objective functions simultaneous in sequence of steps similar to those used in simplex algorithm.

Introduction

Most economic policy problems can be divided into two related problems viz., the "behavioral" problem of forecasting the economy's reactions to policy changes and the "policy" problem proper, of selecting among the alternative possible outcomes. The first problem is sometimes called the descriptive or positive problem, and the second is the prescriptive or normative problem. In the policy context, both normative and positive optimization models are relevant for the behavioral and sub problems respectively.

Aims and Terminology

The approach followed is a direct one, based on problem structure and associated algorithm called Multi-level programming. The algorithm handles two objective functions simultaneous in sequence of steps similar to those used in simplex algorithm. It maximizes one function subject to the other one also being at a maximum. The domain of the maximization of the function must differ or else the objective functions may over rules the other.

Before formally defining multi-level programming, we consider a simple verbal example which helps define the terminology to be used subsequently. In the context of short and medium-term economic policy, a tax reduction sometimes is considered to promote employment objectives. In this case, there are three relevant classes of variables: policy instruments (tax rates), the impact or target variable (the employment level), and behavioral variables not under the direct control of policy makers, but under the control of decentralized individuals and groups (capital spending by firms, consumer spending, etc.). The instruments influence the impact variable(s) only indirectly, by acting on some of the behavioral variables.

Since it is a two-level example, there are two objective functions: a policy objective function which defines preferences at the aggregate level among employment and other, possibly competing, impact variables; and a behavioral objective function, which drives the normative model to yield the kind of market equilibrium that is felt to be most realistic.

Multi-Level Programming Defined

The essence of the multi-level programming problem lies in the domain distinction made above: policy makers are attempting to maximize a policy objective function, while controlling only a subset of the variables. In the two-level case, multi-level programming may be stated as follows:

Find a vector $X = (X_0, X_1, X_2)$

$f_2 = \max (c_2^1 x_0)$

Subject to, $f_1 = \max (c_1^1 x_1)$

$A_{11}X_1 + A_{12} X_2 = b$ .... (3)

$- IX_0 + A_{21}X_1 + A_{22} X_2 = 0$ .... (4)

$X \geq 0$ .... (5)

Where now,

X_0 is a vector of impact variables,

X_1 is a vector of behavioral variables, and

X_2 is a vector of policy variables.

This partitioning is meant to represent a fairly common situation in which:

- Only the impact variables X_0 affect the policy makers "objective function";
- Only the behavioral choice variables X_1 affect the behavioral objective function;
- A_{11} is a technological matrix of resource requirements;
- The matrix A_{12} expresses the effect of the policy variables X_2 on resource availability (a policy which increases resource availability, such as investment in new irrigation supplies, is represented by a negative element of A_{12});
- The vector b represents the level of resource availability prior to policy intervention;
- A_{21} is a matrix of the effects of the behavioral variables X_1 on the impact variables X_0; and
- A_{22} is a matrix of the direct effects of the policy variables X_2 on the impact variables X_0 (in many cases this matrix would be zero and so policies would have to achieve their impacts indirectly, viz., through the matricesA_{12} and A_{21}).

As for linear programming, any non negative vector X is called a solution. Any solutions satisfying (3) - (5) are called a (primal) feasible solution. Any solution maximizing (2) for given X_2 is called a behavioral optimal solution. Any behavioral optimal solution maximizing (1) is called a policy optimal solution.

For a given level of X_2, (2) – (5) define a linear programming problem. However (1) through (5) is not a linear programming problem; hence there is a need for algorithm as defined in (2) multi level programming is not restricted to linear or continuous functions.

It encompasses multiple levels of optimization, multilevel programming constitute a generalization of mathematical programming. The sense in which the later is a special case of multilevel programming may be seen in the following problem definitions:

Mathematical Programming, P_1

A mathematical programming problem P_1 may be written in general as;

Find x such that,

$f(x) \rightarrow \max$ .... (6)

Subject to, $g_i(x) = 0, i = 1, 2, ... m$ .... (7)

Multilevel Programming, P_2

A Multilevel programming problem P_2 may be written in general as;

Find, x_j, $j = 1, 2, ..., n$ such that

$f_j(x_j / x_k), \quad k = j + 1, ..., n) \rightarrow \max, \quad j = 1,2,.........,n$ .... (8)

Subject to, $g_i(x_j, \ j = 1,2,.........,n) = 0, \ i = 1,2,......,m$ .... (9)

Mathematical programming (P_1), can be seen as a special case of Multilevel programming (P_2), since if n = 1, then P_2 becomes:

Find x_1 such that,

$f_1(x_1) \rightarrow \max$ .... (10)

Subject to, $g_i(x_1) = 0, i = 1, 2, ..., m$ .... (11)

Which is P_1, mathematical programming.

If n = 2, then P_2 becomes the following problem:

Find x_1, x_2 such that,

$f_2(x_2) \rightarrow \max$ .... (12)

Subject to, $f_1(x_1 / x_2) \rightarrow \max$ .... (13)

and, $g_i(x_1, x_2) = 0, i = 1, 2, ..., m$ .... (14)

This last problem is multilevel programming in the two level case, as spelled out above. The problem P_2 is multilevel programming in general.

Related Methods

The extensive multilevel planning literature, mainly associated with the names Dantzig and Wolfe and Malinvaud has addressed the problem of two objective functions at the policy and behavioral levels. However, each of these treatments has been concerned with a specification which could in principle, be represent as a single large scale mathematical programming problem. For various reasons including the lack of full information at the outset, solution procedures have been sought through a sequence of mathematical programs.

Perhaps the closest affinity to Multi-level programming is found in control theory, where a policy objective is maximized via choice of values of instrument variables (controls), but where additional variables (state variables) are not directly controlled. The policy objective typically is defined over a subset of state variables.

The clearest description of the policy problem as a Multi-level problem with normative choices at the policy level is found in Theil [15, 372-87]. As Theil suggest game theory may also express the interaction between decisions at the two levels. More specifically, the Stackelberg game can be seen to be a special case of Multi-level programming. In (8) – (9) above, the Stackelberg game is the case in which n = 2, the objective function f_2 represents the behavior of the "leader" and f_1 represents the behavior of the "follower". However there is a very important procedural difference in that the solution of a game requires explicit knowledge of "reaction functions" whereas they are allowed to be implicit in the activity analysis format of multilevel programming.

Conclusion

The separation of policy problems into two components has long been accepted as a rational approach. However this approach has not been implemented systematically. This paper does not describe a substantial numerical test application of multilevel programming algorithm as this technique is overly long.

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